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RAN-2203000205023003

T.Y.B.Sc. (Sem. V) Examination October - 2023

MTH-503 Real Analysis I

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(૧) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **T.Y.B.Sc. (Sem. V)**

Name of the Subject :

☛ **MTH-503 Real Analysis I**

Subject Code No.: **2203000205023003**

Seat No.:

Student's Signature

(2) Figures to the right indicate marks of the question.

(3) Follow usual notations and conventions.

Q. 1 **Answer any FIVE from the following.** **[10]**

1. Define Bounded sequence of real numbers and give an illustration of a non bounded sequence.
2. Check if the sequence $\{\tan n\}_{n=1}^{\infty}$ is monotone or not ? Justify your answer.
3. Write statement of "Nested interval theorem".
4. Define limit inferior for a bounded below sequence of real numbers.
5. Define (i) Alternating series (ii) Harmonic series.
6. Define convergence and conditional convergence of a series of real numbers.
7. Give statement of the "ROOT TEST" for the absolute convergence of the series of real numbers.
8. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.

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Q. 2 **Answer any TWO.** **[10]**

(a) Prove that every convergent sequence of real numbers is bounded.

(b) If $\{s_n\}_{n=1}^{\infty}$ is a bounded sequence of real numbers, and $\{t_n\}_{n=1}^{\infty}$ converges to 0, then prove that $\{s_n \cdot t_n\}_{n=1}^{\infty}$ also converges to 0.

(c) Let $s_1 = \sqrt{2}$ and let $s_{n+1} = \sqrt{2} \cdot \sqrt{s_n}$ for $n \geq 2$. Prove that $\{sn\}_{n=1}^{\infty}$ is convergent.

Q. 3 **Answer any TWO.** **[10]**

(a) If $\{s_n\}_{n=1}^{\infty}$ the sequence of real numbers converges, then prove that $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

(b) Let $\{s_n\}_{n=1}^{\infty}$ diverges to infinity and if $\{t_n\}_{n=1}^{\infty}$ is bounded then prove that $\{s_n + t_n\}_{n=1}^{\infty}$ diverges to infinity.

(c) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers and for each n $\in I$, let

$s_n = a_1 + a_2 + a_3 + ... + a_n$

$t_n = |a_1| + |a_2| + |a_3| + ... + |a_n|$

Prove that if $\{t_n\}_{n=1}^{\infty}$ is a Cauchy sequence, then so is $\{s_n\}_{n=1}^{\infty}$.

Q. 4 **Answer any TWO.** **[10]**

(a) Define a series of real numbers and if $\sum_{n=1}^{\infty} a_n$ is a series of nonnegative numbers $s_n = a_1 + a_2 + a_3 + ... + a_n$ $n \in I$. Then prove that

(i) $\sum_{n=1}^{\infty} a_n$ converges if the sequence $\{s_n\}_{n=1}^{\infty}$ is bounded.

(ii) $\sum_{n=1}^{\infty} a_n$ diverges if the sequence $\{s_n\}_{n=1}^{\infty}$ is not bounded

(b) Check the convergence of the series $1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots$

(c) Prove that the series $(1 - 2) - (1 - 2^{\frac{1}{2}}) + (1 - 2^{\frac{1}{3}}) - (1 - 2^{\frac{1}{4}}) + \dots$ converges.

Q. 5 Answer any TWO. [10]

(a) Prove that:

(i) If $\sum_{n=1}^{\infty} |b_n| < \infty$ and if $\lim_{n \to \infty} \left|\frac{a_n}{b_n}\right|$ exists, then $\sum_{n=1}^{\infty} |a_n| < \infty$

(ii) If $\sum_{n=1}^{\infty} |a_n| < \infty$ and if $\lim_{n \to \infty} \left|\frac{a_n}{b_n}\right|$ exists, then $\sum_{n=1}^{\infty} |b_n| = \infty$

(b) For what values of x does the series $\sum_{n=2}^{\infty} \frac{1}{n (\log n)^{x}}$ converge ?

(c) Using appropriate test of convergence check the convergence for the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
